Classroom Poster Inside

Grades 6–8

Expect the Unexpected With Math®

A Program of The Actuarial Foundation

The Power of Probability

Poster/Teaching Guide for Grades 6–8

Aligns with NCTM and Common Core State Standards

Join Athena and Rick as they solve real-world problems through powerful mathematical thinking!

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What is an actuary? An actuary is an expert in statistics who works with businesses, governments, and organizations to help them plan for the future. Actuarial science is the discipline that applies math and statistical methods to assess risk.

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Bonus:
Includes Online Student Probability Challenge

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Dear Teacher:

Welcome to **The Power of Probability**, a new math program aligned with NCTM and Common Core State Standards, which gives students opportunities to practice their skills and knowledge of the mathematics of probability.

Developed by The Actuarial Foundation, the program's lessons and worksheets motivate students through activities that use mathematics for real purposes. We hope you enjoy this new program!

Sincerely,
The Actuarial Foundation

Overview/Getting Started:

This program is designed to supplement your existing instruction of these **probability** topics:

- strategies to identify favorable and total outcomes;
- calculating simple probability;
- sampling and proportions;
- calculating compound probability; and
- using probability concepts to solve real-world problems.

The materials are taught through this **story line:** Athena and Rick are two middle-school students admired for their powerful mathematical thinking and real-world problem-solving skills. Asked by classmates for help preparing for a mathematics standardized test, the duo develops a smartphone application with practice math problems. The app, featuring a fictitious celebrity couple and their pet, is so successful that Athena and Rick decide to form a business, which provides them with new opportunities and challenges.

The program includes **three lessons**, each with a corresponding **worksheet**. A **bonus worksheet**, which also promotes financial literacy, gives students the opportunity to apply the skills and knowledge they've learned in a real-world context.

A **new bonus online challenge** engages students to use probability skills to help Rick and Athena plan a summer concert tour. This online activity, which allows students to use math for real purposes, is designed as an optional assignment for in or out of the classroom.

A **classroom poster** displays strategies for identifying outcomes as well as determining probability, and underscores the concept that probability can be expressed as a fraction, a decimal, or a percentage.

Lesson 1 Probability Basics

OBJECTIVES:
Students will—

- understand that probability can be expressed as a fraction, a decimal, or a percentage; and
- use tree diagrams, tables, and the fundamental counting principle to calculate probability.

Time Required: 30 minutes, depending on how much review the class needs, plus additional time for the worksheet

Materials:

- one coin (for demonstration)
- Worksheet 1: "Smartphone Test Prep"

DIRECTIONS:

1. Show the class a coin. Ask the class whether it will land on heads or tails if flipped. Students should answer that it could land on either heads or tails. Ask if there is a way to quantify the chance that it will land on heads. If the class doesn't mention the word *probability*, introduce it and note that it means "a fraction, decimal, or percentage describing the likelihood of an event occurring." Explain how no event can have less than a 0% chance or more than a 100% chance of occurring.
2. Ask for examples of how probability is used in the real world. The topic of weather forecasts may be mentioned. Make sure the class understands that a 40% probability of precipitation means that there is a 40% likelihood that precipitation will fall within a given area. Gaming/odds may also be mentioned. You can also introduce to students that companies (insurance and financial companies in particular), statistical experts such as actuaries, and individuals in daily life use probability to make reasonable predictions about the future and to assess risk.
3. Ask what the probability is of a flipped coin landing on heads (1/2). Ensure that the class understands that the numerator (1) represents the number of favorable outcomes (heads) while the denominator (2) represents all possible outcomes (heads and tails). If students haven't mentioned it, ensure that they are also able to express the probability as .5 or 50%.
4. Ask what outcomes (i.e., heads/tails combinations) are possible if the coin is flipped two times, e.g., heads/heads or tails/heads. If students begin to offer outcomes in a haphazard order, ask them how they could make sure they recorded all possible outcomes without double counting. Suggest, for example, a tree diagram and model how it can be used to identify the four different outcomes. Ask what the probability is for any one outcome (1/4, .25, or 25%).
5. Model for students how a table or an organized list could be used to determine the number of possible outcomes. See poster front for example if necessary.

Lesson 1 Probability Basics *continued*

6. Ask what the probability is of flipping one head and one tail. There are four possible outcomes and two favorable ones (heads/tails and tails/heads). Point out that even though this would initially be depicted as 2/4, we would want to reduce to lowest terms, so our final answer would be 1/2, .5, or 50%.
7. Now ask how many outcomes are possible if the coin were flipped three times (eight outcomes). Ask students to explain how a tree diagram, a table, or an organized list could be used to solve the problem in an organized way. Demonstrate if necessary.
8. Ask students if there is a way to determine the number of outcomes without using a table or a tree diagram. Demonstrate how the fundamental counting principle could be used, i.e., two possible outcomes for the first, second, and third flips or 2 x 2 x 2 = 8 or 2^3.
9. Distribute **Worksheet 1**. Ensure that students are comfortable using all three probability methods. Review answers as a class.

Lesson 2 Simple Probability and Sampling

OBJECTIVES:
Students will—

- understand that proportions can be used to make predictions about a population based on a sample;
- identify the difference between outcomes and events; and
- add the probabilities of the outcomes that are part of an event to determine the probability of an event.

Time Required: 30 minutes, depending on how much review the class needs, plus additional time for the worksheet

Materials:

- one six-sided die (for demonstration)
- calculators
- Worksheet 2: "A Call for Assistants"

DIRECTIONS:

1. Show a single six-sided die to the class. Ask what the possible outcomes are. Record the outcomes on the board. Ask the class to calculate the probability of each outcome as a fraction (1/6 for each). Indicate that when the probabilities of all individual outcomes are added together, the sum is 1.
2. Ask what the probability of rolling an even number is. Students should indicate that the probability is 1/2 or 50%. They may calculate this by writing the number of favorable outcomes (3) over the total number of outcomes (6) to get 3/6, which should be reduced to lowest terms (1/2). Show how they could also determine this by adding the probability of each even outcome (1/6 + 1/6 + 1/6) to arrive at 3/6 or 1/2 or 50%.
3. Explain that the even outcomes here are 2, 4, or 6. Rolling an even number is called an event. The sum of the even outcomes equals the probability of an event. If necessary, repeat with other examples, such as the probability of rolling a number other than a 3, the probability of rolling a number less than 3, etc.
4. Distribute **Worksheet 2** and calculators. Read the introduction and review the facts with the class.
5. Ask students to complete the worksheet. Explain that the bonus question requires them to apply what they learned about probability in Lesson 1. Review answers as a class. Make sure that the review includes a discussion of how proportions are used to make predictions about the population as a whole by using the results of a poll of a sample of the total customer base (e.g., 12/2,000 = x/50,000) and the validity of doing so.

Lesson 3 Compound Probability

OBJECTIVES:
Students will—

- use a tree diagram to derive the formula for compound probability; and
- use the formula for compound probability to calculate the probability of multiple independent events.

Time Required: 20–30 minutes, depending on how much review the class needs, plus additional time for worksheets

Materials:

- coin (for demonstration)
- Worksheet 3: "Math Masters"

DIRECTIONS:

1. Show the coin to the class and ask what is the probability of the coin landing on heads.
2. Recall the work done in Lesson 1 on tree diagrams and ask for a volunteer to explain how all the outcomes for three flips in a row could be depicted.
3. Ask what the probability of a coin landing on heads three times in a row would be. Walk the students through a tree diagram to demonstrate that there is one outcome for three heads in a row while there are seven unfavorable outcomes (HHT, HTT, HTH, TTT, TTH, THT, THH), so the probability is one out of eight or 1/8 or 12.5%.

Smartphone Test Prep

Athena and Rick, two middle-school students frequently called upon for their powerful math skills, have been flooded with numerous student requests for help with the dreaded upcoming mathematics standardized test. "Wow, how are we going to get to everyone in time?" wondered Athena.

Rick had a flash: How about a smartphone app that provides our classmates with fun practice problems? The pair got to work on their first set of math problems.

WORK THE MATH (Show your work on separate paper.)

1. Lee and Lily Limelight, the famous celebrity couple (famous for what, we don't know!), love to dress Iggy, their pet iguana, for photographers. Iggy's hats include a diamond tiara, a leather biker cap, a silk jester's hat, and a bejeweled pith helmet. Iggy's footwear consists of hot pink high-top sneakers, rhinestone work boots, and three-inch platform shoes. Using a **table**, determine how many outfits the Limelights can make for Iggy, assuming they pick one hat and one set of footwear.

2. Lee decides to purchase two vests for Iggy, one chartreuse and the other teal. Using a **tree diagram**, determine how many outfits he could make with one hat, one vest, and one set of footwear. (You can use the tree diagram on the classroom poster as an example.)

3. Lily then purchases a little bling for Iggy: one silver necklace and one gold necklace. Using the **fundamental counting principle**, determine the number of outfits she could make with one of each category of clothing, including footwear, one hat, one vest, and one necklace.

NOW TRY THIS

If the Limelights reach into Iggy's closet and pick one complete outfit at random, what is the probability that the outfit will include a tiara, high-tops, teal vest, and silver necklace?

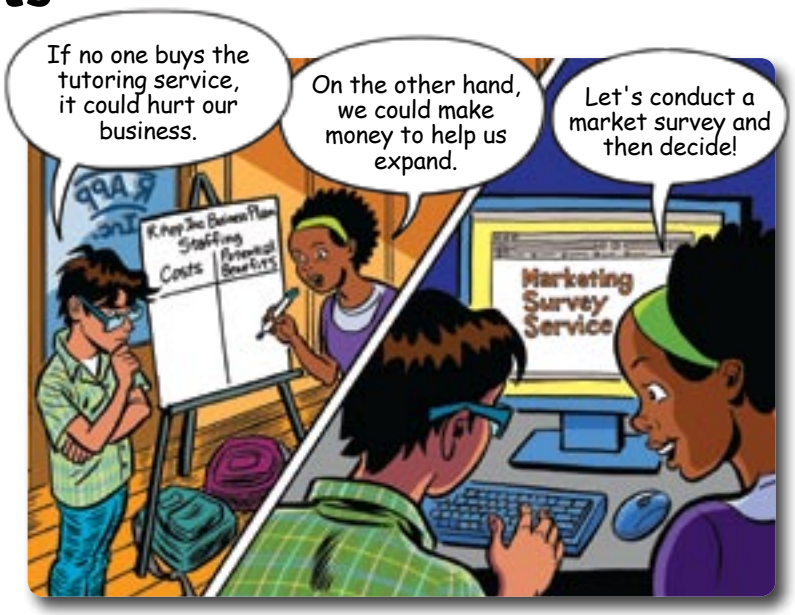
Hint: Think about probability as the number of outcomes meeting the requirements out of the total number of equally likely outcomes.

A Call for Assistants

Athena and Rick's app is a huge hit, so they form a company: R App Inc. Some customers ask for a new service: math tutoring.

After estimating that it will cost $14,000 a year to hire tutors, the duo wonders what to do. "If no one buys the service, the additional cost might sink us," fretted Rick. Athena responded, "But if we do nothing, we could lose income that could help R App grow."

To make an informed decision, Athena suggests doing a market survey. The pair decides to survey 75 randomly selected users (out of 3,750 users) to find out how much additional revenue they can expect.



WORK THE MATH (Show your work on separate paper.)

Amount of Revenue	Probability
$1,000	10% or 1/10
$7,000	10% or 1/10
$14,000	10% or 1/10
$20,000	20% or 1/5
$30,000	25% or 1/4
$50,000	25% or 1/4

1. What percentage of the population is the sample?
2. What is the probability that the new service will make money?
3. What is the probability that the new service will lose money?
4. Why don't your answers to the first two questions add up to 100%?
5. What do you think Athena and Rick should do? Explain your thinking.

NOW TRY THIS

Sampling their customer base gives Rick and Athena an idea for new problems for the app:

The Limelights are negotiating with Lowest Common Denominator Cable to produce a reality show starring (you guessed it!) the Limelights. The cable company has 50,000 customers. To make sure there are enough viewers, the cable company surveyed 2,000 customers. Twelve customers said they would watch "Life With the Limelights." When asked if they would watch a show featuring Iggy the iguana, the number spiked to 1,724 customers.

1. How many customers do you predict would watch "Life With the Limelights"?
2. How many customers do you predict would watch a show starring Iggy?

Math Masters

After only six months, R App is making a sizable profit. "We should donate some of our profits to a worthy cause," Athena thinks aloud. Rick replies, "How about a donation to our school because they made us the mathematicians we are today?" "Our gift might inspire others to excel in math, too," chimes in Athena.

After considering gifts like a random lunch menu generator for the cafeteria and a statue of Isaac Newton, the grateful duo donates a scholarship to the first contestant to correctly answer four challenging problems in a row in a math competition.

WORK THE MATH (Show your work on separate paper.)

For the first scholarship competition, there are three contestants:

Contestant	Average Math Test Score
Contestant A	95%
Contestant B	90%
Contestant C	80%

1. What is the probability that each of the contestants will answer four questions in a row correctly?

Contestant A _____ Contestant B _____ Contestant C _____

Hint: You can assume the probability of a contestant answering a single question correctly equals his or her average test score.

2. If Athena and Rick decided to make the competition more difficult by adding a fifth question, what would the probability of winning be for each contestant?

Contestant A _____ Contestant B _____ Contestant C _____

3. What is the probability of all three contestants answering four questions in a row correctly?

Working with compound probability for the contest gave Athena and Rick ideas for new problems for the app.

Iggy the iguana has become a breakout reality star! Iggy's agent has successfully negotiated an appearance on the hit show "You Want Me to Do What?!" To win the grand prize for the Love a Lizard Foundation, Iggy must complete three wacky but dangerous challenges: Dodge the Doberman (75% probability of winning), Root Canal (80% probability of winning), and Is It Spoiled? (90% probability of winning). What is the probability of Iggy winning the grand prize?

Mind Your Own Business!

R App has been in business for a while and Athena and Rick have turned a profit. The two have always saved some of the money they made for extra things they wanted. "It would be nice to spend this money, but it's also smart to invest it to make even more money," Athena mentions. "But how can we predict if an investment is risky? I certainly don't want to lose any money!" replied Rick.

The pair decides to email their questions to Joe, an actuary, whose daily work involves using statistics to help predict risk. Joe suggests that Rick and Athena apply their knowledge of mathematical probability to help them make smart investment choices.

WORK THE MATH (Show your work on separate paper.)

Rick and Athena discover thousands of investment choices, but they limit their research to the following:

- A one-year bank certificate of deposit paying 1% simple interest annually. (The U.S. government protects the value of these accounts, and historically, no one has ever lost their money, even if the bank goes out of business.)
- Shares in a mutual fund, representing an investment in the stocks of the largest U.S. companies. Over the past 20 years, this has increased an average of 7%. In the past 20 years, the increase has been over 10% eleven times and the decrease has been over 10% three times.
- A share of a new company developing an app to chart the quickest path through school hallways. Rick estimates that there is a 99% chance that the company will not make money.

Suppose Athena has $1,000 to invest:

1. What is the probability that the certificate could become worthless?
2. What is the probability that the mutual fund will lose more than 10% of its value in the next year?
3. What is the probability that the start-up company will become successful?

NOW TRY THIS

Making reasonable predictions about the future gives Athena and Rick another app idea:

Because Iggy is so popular, the Limelights come up with a money-making idea to build upon his star status. They collect a few of his scales and plan to clone Iggy replicas to sell to the public for outrageous prices. The Limelights' investment advisor estimates that a $100,000 investment will be necessary, and that the business has a 10% chance of making $1,000,000 but a 90% chance of going bankrupt. What should the Limelights do?

4. Ask if there is a way to calculate the probability of tossing three heads in a row without setting up the tree diagram. If necessary, point out that the probability of one heads flip is 1/2 or 50% or .5 and, for three heads in a row:

   1/2 x 1/2 x 1/2 = 1/8 *or*
   .5 x .5 x .5 = .125 *or*
   50% (.5) x 50% (.5) x 50% (.5) = 12.5% or .125

   If the class is familiar with exponents, demonstrate that the probability of any number of heads in a row equals $1/2^{(\text{number of flips})}$.
5. On the board, write the formula for compound probability, i.e., the probability of two independent events: P(AB) = P(A)·P(B). Explain the formula and that A and B are *independent events*: the probability of A occurring does not affect the probability of B occurring, and vice versa.
6. Provide an example or two to show how the formula works. For example, if the probability of rain is 20% on Saturday and 50% on Sunday, what is the probability of rain on both days? (20% times 50% = 10%). Demonstrate how this could be done with fractions (2/10 times 1/2 = 2/20 which reduces to 1/10) or decimals (.2 times .5 = .10). To reinforce the abstract concept of compound probability, use a tree diagram to demonstrate how rain on both days represents one out of 10 possible outcomes.
7. Distribute **Worksheet 3**. Read the introduction and review the facts with the class. Ask students to complete the worksheet. Review answers as a class.

Additional Free Math Resources from The Actuarial Foundation

www.actuarialfoundation.org/programs/for_teachers.shtml and

www.scholastic.com/unexpectedmath

New **Student Online Probability Challenge**

In this online activity, students are challenged to use probability skills to help Rick and Athena plan a summer concert tour. This activity allows students to use math for real purposes, and is designed as an optional assignment for in or out of the classroom.

Expect the Unexpected With Math® Series:

- **Converting decimals, fractions, and percents:** *Conversions Rock*
- **Graphing:** *Bars, Lines, & Pies*
- **Perimeter, area, surface area, and volume:** *Setting the Stage with Geometry*
- **Pre-Algebra:** *Solving the Unknown with Algebra*
- **Probability:** *Shake, Rattle, & Roll*

Also Available:

- **The Math Academy Series: Using Math in the Real World**
- **Building Your Future: A Financial Literacy Series**
- **Math Grant Opportunities**

Worksheet Answer Key

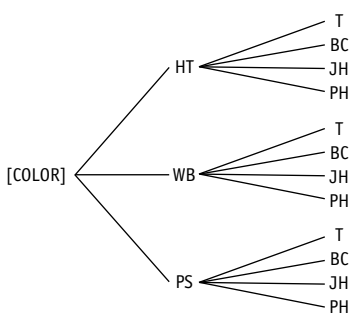
Worksheet 1: "Smartphone Test Prep"

1. 12 outfits:

HT T	HT BC	HT JH	HT PH
WB T	WB BC	WB JH	WB PH
PS T	PS BC	PS JH	PS PH

Key: HT (high-tops); BC (biker cap); T (tiara); JH (jester's hat); WB (work boots); PH (pith helmet); PS (platform shoes)

2. 24 outfits, comprising two trees, one for chartreuse and one for teal:



3. 48 outfits (2 necklaces x 2 vests x 3 footwear x 4 headgear)

Now Try This: The probability of any one outfit being selected is 1/48 or 2.1% (rounded to the nearest tenth of a percent).

Worksheet 2: "A Call for Assistants"

1. The sample is 2% of the population (75/3,750).
2. 70% (the sum of the probabilities for revenues of $20,000, $30,000, and $50,000).
3. 20% (the sum of the probabilities for revenues of $1,000 and $7,000).
4. There is also a 10% probability that the new service will break even.
5. Answers will vary but might include that any risk of losing money is unacceptable or that the potential profits are too good to pass up. Although there is no right answer, the students' tolerance for risk should be addressed as they defend their answers.

Now Try This:

1. 300 viewers (50,000/2,000 x 12)
2. 43,100 viewers (50,000/2,000 x 1,724)

Worksheet 3: "Math Masters"

1. Contestant A: 81.5% (.95 x .95 x .95 x .95)
   Contestant B: 65.6% (.9 x .9 x .9 x .9)
   Contestant C: 41.0% (.8 x .8 x .8 x .8)
2. Contestant A: 77.4% (.95 x .95 x .95 x .95 x .95)
   Contestant B: 59.0% (.9 x .9 x .9 x .9 x .9)
   Contestant C: 32.8% (.8 x .8 x .8 x .8 x .8)
3. 21.9 % (.815 x .656 x .41)

Now Try This: 54.0% (.75 x .80 x .90)

Bonus Worksheet: "Mind Your Own Business!"

1. Investment analysts generally consider FDIC-insured accounts to be risk free, so the risk would be 0%. The U.S. government would have to fail for the certificates to become worthless.
2. 15% (3/20)
3. 1%

Now Try This: Answers will vary, but should indicate that the investment is highly risky with a small probability of a significant profit.